With today's post, let's wrap up arrangements for the time being. We will discuss some complex circular arrangement constraints (which we will easily work through) today and start with combinations (i.e. picking "r" units out of "n" units) next week. Thereafter we will look at questions involving both, picking and arranging (yeah, that will be fun!).

Question 1: A group of 8 friends sit together in a circle. If A refuses to sit beside B unless C sits on the other side of A as well, how many possible seating arrangements are possible?

Solution: Let's start with what we know. We know that the total number of ways in which 8 people can be arranged around a circular table is (8-1)! = 7!

Since we do not want A to sit next to B, let's try and make them sit together. This will give us the number of arrangements that are unacceptable to us. Let's say that A and B are a single unit. So now there are 7 units which need to be arranged in a circle. This can be done in (7-1)! = 6! ways. Since there are two arrangements possible, AB and BA, within the unit, we need to multiply 6! by 2.

Number of arrangements in which A and B sit together = 2*6!

We can subtract these 'unacceptable arrangements' from total arrangements to get the number of 'acceptable arrangements'. But this number of 'unacceptable arrangements' includes those arrangements where C is sitting on the other side of A. But those arrangements are acceptable to us so we should not subtract them out. How many such arrangements are there in which A and B are sitting together and C is sitting beside A too?

Now C, A and B form a single unit leaving us with 6 units to be arranged in a circle. 6 units can be arranged in (6-1)! = 5! ways

CAB can also be arranged as BAC, hence the 5! needs to be multiplied by 2. (Mind you, we will not consider ABC, ACB etc here since A should be in the middle)

Number of arrangements in which A and B sit together and C sits beside A = 2*5!

Therefore, number of unacceptable arrangements = 2*6! – 2*5!

We subtract these out of the total number of arrangements and we get the total number of acceptable arrangements.

Possible number of seating arrangements = 7! – (2*6! – 2*5!) = 3840

If you are wondering about the 'painful' calculation involved in the step above, don't worry. Calculations with factorials are generally quite straight forward.

7! – (2*6! – 2*5!) = 7! – 2*6! + 2*5!

= 2*5! (21 – 6 + 1) (Take 2*5! common out of the three terms)

= 2*120*16 = 32*120 = 3840

I hope the solution makes sense to you. Let's look at another tricky circular arrangement problem.

Question 2: Seven men and seven women have to sit around a circular table so that no two women are together. In how many different ways can this be done?

Solution:

There are 7 men: Mr. A, Mr. B, Mr. C, Mr. D …
and 7 women: Ms. A, Ms. B, Ms. C, Ms. D …

Let’s say we have 14 identical chairs around the round table.

We need to seat the 7 women such that no two of them are together i.e. there should be a man on either side of every woman. Since there are exactly 7 men, the women and men should sit alternately. Let’s make the women sit first. For the first woman who sits, each seat is identical so she sits in one way (say Ms.C takes a seat). Now each seat is distinct relative to this woman (Ms. C). There are 7 seats identified for men (e.g. seats right next to Ms. C and every alternate seat) and 6 for the remaining 6 women. The 7 men can occupy the 7 distinct seats in 7! ways and the 6 women can occupy the 6 distinct seats in 6! ways.

Total number of arrangements = 6!*7!

Something to ponder upon: The total number of arrangements is not 13!. Why?

Question 3: Find the number of ways in which four men, two women and a child can sit at a circular table if the child is seated between the two women.

Solution:

We have 7 people and 7 seats around a circular table.

First let’s make the child sit anywhere in one way since all the places are identical. The two women can sit around the child in 2! ways. Now we have 4 distinct seats (relative to the people sitting) left for the 4 men and they can occupy the seats in 4! ways.

Total number of arrangements = 1*2!*4! = 48

Things to ponder upon:

Case 1: Same question as above but the chairs are numbered i.e. all the seats are distinct. Find the number of ways in which four men, two women and a child can sit around a circular table with numbered seats if the child is seated between the two women.

Case 2: Same question as above but they need to stand in a row instead. Find the number of ways in which four men, two women and a child can stand in a row if the child is standing between the two women.

Are the two cases above equivalent?